Nonsupersymmetric gauge coupling unfication in $[SU(6)]^4 \times Z_4$ and proton stability

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Abstract.We systematically study the three family extension of the Pati-Salam gauge group with an anomaly-free single irreducible representation which contains the known quarks and leptons without mirror fermions. In the context of this model we implement the survival hypothesis, the modified horizontal survival hypothesis, and calculate the tree level masses for the gauge boson and fermion fields. We also use the extended survival hypothesis in order to calculate the mass scales using the renormalization group equation. The interacting Lagrangean with all the known and predicted gauge interactions is explicitly displayed. Finally the stability of the proton in this model is established.

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1. Introduction

The renormalizability of the original Pati-Salam[1] model for unification of flavors and forces rests on the existence of conjugate or mirror partners of ordinary fermions. Mirror fermions are fermions with quantum numbers with respect to the Standard Model (SM) gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ identical to those of the known quarks and leptons, except that they have opposite handedness from ordinary fermions. Their existence vitiate the survival hypothesis [2] according to which chiral fermions that can pair off while respecting a symmetry will do so, acquiring masses grater than or equal to the mass scale of that symmetry.

Today we know how to cancel anomalies without introducing unwanted mirror fermions. As a matter of fact, the three family extension of the Pati-Salam model without mirror fermions was presented recently in the literature, with some aspects of the model briefly analyzed in the original reference[3]. But a systematic analysis of this model is still lacking. In what follows we do such analysis, paying special attention to the implementation of the survival hypothesis [2] and of the modified horizontal survival hypothesis [4]. (For a technical explanation of the terminology used in this article see Appendix A.)

The model under consideration unifies non-gravitational forces with three families of flavors, using the gauge group

$$G \equiv SU(6)_L \otimes SU(6)_R \otimes SU(6)_{CR} \otimes SU(6)_{CL} \times Z_4$$

where $\otimes$ indicates a direct product, $\times$ a semidirect one, and $Z_4 \equiv (1, P, P^2, P^3)$ is the four-element cyclic group acting upon $[\mathrm{SU}(6)]^4$ such that if (A,B,C,D) is a representation of $[\mathrm{SU}(6)]^4$ with A a representation of the first factor, B of the second, C of the third, and D of the fourth, then P(A,B,C,D)=(B,C,D,A) and then Z_4(A,B,C,D)$\equiv$(A,B,C,D) $\oplus$

(B,C,D,A) $\oplus$ (C,D,A,B) $\oplus$ (D,A,B,C). The electric charge operator in G is defined as[3]

$$Q_{EM} = T_{ZL} + T_{ZR} + [Y_{(B-L)_L} + Y_{(B-L)_R}]/2, \tag{1}$$

where $(B-L)_{L(R)}$ stands for the local Abelian factor of $(Baryon - Lepton)_{L(R)}$ hypercharge associated with the diagonal generators $Y_{(B-L)_{L(R)}} = Diag(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, 1, -1)_{L(R)}$ of $\mathrm{SU}(6)_{CL(CR)}$

The irreducible representation (irrep) of G which contains the known fermions is

$$\psi(144) = Z_4\psi(\bar{6}, 1, 1, 6) = \psi(\bar{6}, 1, 1, 6) \oplus \psi(1, 1, 6, \bar{6}) \oplus \psi(1, 6, \bar{6}, 1) \oplus \psi(6, \bar{6}, 1, 1).$$

The model described by the structure $[G, \psi(144)]$ is a grand unification model which contains the three family SM gauge group, the three family left-right symmetric extension of the SM[5] $[\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{(B-L)}]$ and the three family chiral color extension of the SM[6] $[\mathrm{SU}(3)_{CR} \otimes \mathrm{SU}(3)_{CL} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y]$. Finally, $[G, \psi(144)]$ is the chiral extension of the vector-color-like model described by [7, 8] $G^V \equiv \mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_C \otimes \mathrm{SU}(6)_R \times \mathrm{Z}_3$ and $\psi^V(108) = \psi^V(\bar{6}, 6, 1) \oplus \psi^V(6, 1, \bar{6}) \oplus \psi^V(1, \bar{6}, 6)$, where $\mathrm{SU}(6)_C$ in G^V is the diagonal subgroup of $\mathrm{SU}(6)_{CR} \otimes \mathrm{SU}(6)_{CL} \subset G$, and $\psi^V(108) \subset \psi(144)$

That $[G, \psi(144)]$ is free of anomalies and does not contain mirror fermions follows from its particle content. To see this we first show that there is a unique way to embed the SM gauge group for three families in [G,ψ(144)][3] and then write the quantum numbers for $\psi(144)$ with respect to the subgroups of the SM which are [the notation designates behavior under ($\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$)]:

$\psi(\bar{6}, 1, 1, 6) \sim 3(3, 2, 1/3) \oplus 6(1, 2, -1) \oplus 3(1, 2, 1)$

$\psi(1, 6, \bar{6}, 1) \sim 3(\bar{3}, 1, -4/3) \oplus 3(\bar{3}, 1, 2/3) \oplus 6(1, 1, 2) \oplus 9(1, 1, 0) \oplus 3(1, 1, -2)$

$\psi(6, \bar{6}, 1, 1) \sim 9(1, 2, 1) \oplus 9(1, 2, -1)$

$\psi(1, 1, 6, \bar{6}) \sim (8{+}1, 1, 0) \oplus 2(3, 1, 4/3) \oplus 2(\bar{3}, 1, -4/3) \oplus (3, 1, -2/3) \oplus (\bar{3}, 1, 2/3) \oplus 5(1, 1, 0) \oplus 2(1, 1, 2) \oplus 2(1, 1, -2)$,

where the ordinary left-handed quarks correspond to 3(3,2,1/3) in $\psi(\bar{6},1,1,6)$, the ordinary right-handed quarks correspond to $3(\bar{3},1,-4/3)\oplus 3(\bar{3},1,2/3)$ in $\psi(1,6,\bar{6},1)$, the known left-handed leptons are in three of the six (1,2,−1) of $\psi(\bar{6},1,1,6)$, and the known right-handed charged leptons are in three of the six (1,1,2) of $\psi(1,6,\bar{6},1)$. The exotic leptons in $\psi(\bar{6},1,1,6)$ belong to the vectorlike representation $3(1,2,-1)\oplus 3(1,2,1)$ (vectorlike with respect to the SM quantum numbers) and the exotic leptons in $\psi(1,6,\bar{6},1)$ belong to the vectorlike representation $3(1,1,2)\oplus 3(1,1,-2)\oplus 9(1,1,0)$, where three lineal combinations of the nine states with quantum numbers (1,1,0) could be identified as the right-handed neutrinos.

$\psi(6,\bar{6},1,1)$ is formed by 36 exotic spin 1/2 Weyl fermions (we call them *nones* because they have zero lepton and baryon numbers), 9 with positive electric charges, 9 with negative (the charge conjugates to the positive ones), and 18 are neutrals; all together constitute a vectorlike representation with respect to the SM.

Also all the particles in $\psi(1,1,6,\bar{6})$ form a vectorlike representation with respect to the SM, where $5(1,1,0)\oplus 2(1,1,2)\oplus 2(1,1,-2)$ stands for nine exotic fermions, five with zero electric charge (*nones*), two with electric charge +1 and the other two with electric charge −1 (spin 1/2 dileptons); $2(3,1,4/3)\oplus 2(\bar{3},1,-4/3)$ refers to two exotic spin 1/2 leptoquarks with electric charge 2/3; $(3,1,-2/3)\oplus(\bar{3},1,2/3)$ refers to one exotic spin 1/2 leptoquark with electric charge −1/3, and the nine states in (8+1,1,0)=(8,1,0)+(1,1,0) (*quaits*)+(*quone*) are the so-called dichromatic fermion multiplets [6] (also *nones*) which belong to the $(3,\bar{3})$ representation of the $SU(3)_{CR}\otimes SU(3)_{CL}$ subgroup of $SU(6)_{CR}\otimes SU(6)_{CL}$.

Notice that contrary to the original Pati-Salam model, the G symmetry and the representation content of $\psi(144)$ forbid mass terms for fermion fields at the unification scale, but according to the survival hypothesis[2] the vectorlike substructures pointed in this section (all the exotic particles in the model) should get masses at scales above M_Z, the

known weak interaction mass scale.

2. The Model

The model under consideration contains 140 spin 1 gauge boson fields, 144 spin 1/2 Weyl fermion fields, and a conveniently large number of spin 0 scalar fields. We use for them the following notation:

2..1 The gauge bosons

For the gauge boson fields we define:

a)-For the 70 gauge fields of SU(6)$_{CL}$ and SU(6)$_{CR}$

$$\mathbf{A}_{CL(CR)} = \frac{1}{\sqrt{2}} \begin{pmatrix} D_1 & G_2^1 & G_3^1 & \widetilde{X}_1 & \widetilde{Y}_1 & \widetilde{Z}_1 \\ G_1^2 & D_2 & G_3^2 & \widetilde{X}_2 & \widetilde{Y}_2 & \widetilde{Z}_2 \\ G_1^3 & G_2^3 & D_3 & \widetilde{X}_3 & \widetilde{Y}_3 & \widetilde{Z}_3 \\ X_1 & X_2 & X_3 & D_4 & P_1^- & P^0 \\ Y_1 & Y_2 & Y_3 & P_1^+ & D_5 & P_2^+ \\ Z_1 & Z_2 & Z_3 & \widetilde{P}^0 & P_2^- & D_6 \end{pmatrix}_{CL(CR)} \tag{2}$$

where

$$\begin{aligned} D_{\delta CL(CR)} &= (G_\delta^\delta)_{CL(CR)} + \sqrt{\frac{1}{30}} B_{(B-L)_{L(R)}} + \sqrt{\frac{2}{15}} B_{1YL(R)}, \quad \delta = 1,2,3; \\ D_{4CL(CR)} &= -\sqrt{\frac{3}{10}} B_{(B-L)_{L(R)}} - \frac{1}{\sqrt{30}} B_{1YL(R)} - \frac{1}{\sqrt{2}} B_{2YL(R)}; \\ D_{5CL(CR)} &= \sqrt{\frac{3}{10}} B_{(B-L)_{L(R)}} - \frac{4}{\sqrt{30}} B_{1YL(R)}; \\ D_{6CL(CR)} &= -\sqrt{\frac{3}{10}} B_{(B-L)_{L(R)}} - \frac{1}{\sqrt{30}} B_{1YL(R)} + \frac{1}{\sqrt{2}} B_{2YL(R)}; \end{aligned}$$

with $(G'^{\delta}_{\eta})_{CL(CR)}$, $\delta, \eta = 1, 2, 3$ the gauge fields associated with SU(3) $_{CL(CR)}$ $(G^1_{1CL(CR)} = B_{1gCL(CR)}/\sqrt{2} + B_{2gCL(CR)}/\sqrt{6}$, $G^2_{2CL(CR)} = -B_{1gCL(CR)}/\sqrt{2} + B_{2gCL(CR)}/\sqrt{6}$, $G^3_{3CL(CR)} = -2B_{2gCL(CR)}/\sqrt{6}$ such that $\sum_\delta (G^\delta_\delta)_{CL(CR)} = 0$, and $B_{1gCL(CR)}$ and $B_{2gCL(CR)}$ are the gauge fields associated with the diagonal generators of $\mathrm{SU}(3)_{CL(CR)}$). $B_{(B-L)_{L(R)}}$ is the gauge boson associated with the generator $Y_{(B-L)_{L(R)}}$, and $B_{1YL(R)}$ and $B_{2YL(R)}$ are two gauge bosons associated with the $\mathrm{SU}(6)_{CL(CR)}$ diagonal generators $Y_{1L(R)} = Diag(2, 2, 2, -1, -4, -1)/\sqrt{15}$ and $Y_{2L(R)} = Diag(0, 0, 0, -1, 0, 1)$ respectively. X_δ, Y_δ and Z_δ are spin 1 lepto-quark gauge bosons with electric charges $-2/3, 1/3$ and $-2/3$ respectively, with $\delta = 1, 2, 3$ a color index. $P^\pm_\kappa, \kappa = 1, 2$; and P^0 are spin 1 dilepton gauge bosons with electric charges as indicated.

b)-For the 70 gauge fields of $\mathrm{SU}(6)_L$ and $\mathrm{SU}(6)_R$

$$\mathbf{A}_{L(R)} = \frac{1}{\sqrt{2}} \begin{pmatrix} A_1 & B'^+_1 & H'^0_1 & B^+_2 & H^0_2 & B^+_3 \\ B'^-_1 & A_2 & B^-_4 & H'^0_3 & B^-_5 & H^0_4 \\ \overset{\sim\prime 0}{H}_1 & B^+_4 & A_3 & B'^+_6 & H'^0_5 & B^+_7 \\ B^-_2 & \overset{\sim\prime 0}{H}_3 & B'^-_6 & A_4 & B^-_8 & H'^0_6 \\ \overset{\sim 0}{H}_2 & B^+_5 & \overset{\sim\prime 0}{H}_5 & B^+_8 & A_5 & B'^+_9 \\ B^-_3 & \overset{\sim 0}{H}_4 & B^-_7 & \overset{\sim\prime 0}{H}_6 & B'^-_9 & A_6 \end{pmatrix}_{L(R)} \tag{3}$$

where the diagonal and the primed entries in Eq.(3) are related to the physical fields as explained in the Appendix B.

2..2 The Fermionic content

For the spin 1/2 Weyl fields we use the following definitions:

$$\psi(\bar{6},1,1,6)_L = \begin{pmatrix} d_1 & d_2 & d_3 & e^-_{11} & e^{0c}_{12} & e^-_{13} \\ u_1 & u_2 & u_3 & -n^0_{11} & n^+_{12} & -n^0_{13} \\ s_1 & s_2 & s_3 & -e^-_{21} & e^{0c}_{22} & e^-_{23} \\ c_1 & c_2 & c_3 & n^0_{21} & n^+_{22} & -n^0_{23} \\ b_1 & b_2 & b_3 & -e^-_{31} & e^{0c}_{32} & -e^-_{33} \\ t_1 & t_2 & t_3 & n^0_{31} & n^+_{32} & n^0_{33} \end{pmatrix}_L \equiv \psi^\alpha_a \tag{4}$$

where the rows(columns) represent color(flavor) degrees of freedom, (u,d,c,s,b,t) are the quark fields with colors $\delta = 1,2,3$ as indicated, (e_{ij}, n_{ij}), $i,j = 1,2,3$ are lepton Weyl fields with electric charge as indicated, the minus signs are phases chosen for convenience, and the upper c symbol stands for charge conjugation.

$$\psi(1,6,\bar{6},1) = \begin{pmatrix} d^c_1 & u^c_1 & s^c_1 & c^c_1 & b^c_1 & t^c_1 \\ d^c_2 & u^c_2 & s^c_2 & c^c_2 & b^c_2 & t^c_2 \\ d^c_3 & u^c_3 & s^c_3 & c^c_3 & b^c_3 & t^c_3 \\ E^+_{11} & -N^{0c}_{11} & -E^+_{21} & N^{0c}_{21} & -E^+_{31} & N^{0c}_{31} \\ E^0_{12} & N^-_{12} & E^0_{22} & N^-_{22} & E^0_{32} & N^-_{32} \\ E^+_{13} & -N^{0c}_{13} & E^+_{23} & -N^{0c}_{23} & -E^+_{33} & N^{0c}_{33} \end{pmatrix}_L \equiv \psi^A_\Delta \tag{5}$$

where the rows (columns) now represent flavor (color) degrees of freedom. The notation we are using with the lepton fields in $\psi(1,6,\bar{6},1)$ unrelated in principle to the lepton fields in $\psi(\bar{6},1,1,6)$ is consistent with the SM quantum numbers for $\psi(\bar{6},1,1,6) \oplus \psi(1,6,\bar{6},1)$ presented in the Introduction. The known leptons $(\nu_e, e^-, \nu_\mu, \mu^-, \nu_\tau, \tau^-)$ and the known quarks are linear combinations of the leptons and quarks in $\psi(\bar{6},1,1,6) \oplus \psi(1,6,\bar{6},1)$, up to mixing with exotics. Our notation is such that $a, b, ..$; $A, B, ...$; $\alpha, \beta, ...$; $\Delta, \Omega, ...$ stand for $\mathrm{SU}(6)_L$, $\mathrm{SU}(6)_R$, $\mathrm{SU}(6)_{CL}$, and $\mathrm{SU}(6)_{CR}$ tensor indices respectively.

For the sake of completeness we also write:

$$\psi(1,1,6,\bar{6}) \equiv \psi^{\Delta}_{\alpha} = \begin{pmatrix} g^1_1 & g^1_2 & g^1_3 & x_r & y_r & z_r \\ g^2_1 & g^2_2 & g^2_3 & x_y & y_y & z_y \\ g^3_1 & g^3_2 & g^3_3 & x_b & y_b & z_b \\ \widetilde{x}_r & \widetilde{x}_y & \widetilde{x}_b & l^0_1 & l^+_1 & l^0_2 \\ \widetilde{y}_r & \widetilde{y}_y & \widetilde{y}_b & l^-_1 & l^0_3 & l^-_2 \\ \widetilde{z}_r & \widetilde{z}_y & \widetilde{z}_b & l^0_4 & l^+_2 & l^0_5 \end{pmatrix}_L \tag{6}$$

where $g^i_j,\ i,j=1,2,3$ are the $(quaits)+(quone)$ spin 1/2 *nones*; x, y and z are the spin 1/2 leptoquarks with electric charges 2/3, $-1/3$ and 2/3 respectively, $l^{\pm}_j, j=1,2$ are spin 1/2 dilepton fields with electric charges as indicated, and $l^0_j, j=1,...5$ are five nones with zero electric charge.

2..3 The Scalar Content

In order to spontaneously break the G symmetry down to SU(3)$_C\otimes$U(1)$_{EM}$, and to implement at the same time the survival hypothesis and the horizontal survival hypothesis, we need to introduce the following rather complicated scalar sector:

First we introduce the scalar fields ϕ_1 and ϕ_2 with Vacuum Expectation Values (VeVs) such that $\langle\phi_1\rangle \sim \langle\phi_2\rangle \sim M$, where

$$\phi_j = \phi_j(900) = Z_4\phi_j(15,1,1,\overline{15}) = \phi^{[a,b]}_{j[\alpha,\delta]} + \phi^{[\alpha,\delta]}_{j[\Delta,\Omega]} + \phi^{[\Delta,\Omega]}_{j[A,B]} + \phi^{[A,B]}_{j[a,b]}$$

$j=1,2$, and [.,.] stands for the commutator of the indices inside the brackets. The VeVs for $\phi_j,\ j=1,2$ are conveniently chosen in the following directions:

$\langle\phi^{[a,b]}_{1[\alpha,\delta]}\rangle = \sqrt{3}M$ for $[a,b]=[4,1]=[2,3]=[5,6]; [\alpha,\delta]=[5,6]$

$\langle\phi^{[\Delta,\Omega]}_{1[A,B]}\rangle = \sqrt{3}M$ for $[A,B]=[4,1]=[2,3]=[5,6]; [\Delta,\Omega]=[5,6]$

$\langle\phi^{[A,B]}_{1[a,b]}\rangle = M$ for $[a,b]=[A,B]=[4,1]=[2,3]=[6,5]$

$\langle\phi^{[\alpha,\delta]}_{j[\Delta,\Omega]}\rangle = 0; j = 1,2$

$\langle\phi^{[a,b]}_{2[\alpha,\delta]}\rangle = \sqrt{3}M$ for $[a,b] = [1,2] = [6,3] = [4,5]; [\alpha,\delta] = [4,5]$

$\langle\phi^{[\Delta,\Omega]}_{2[A,B]}\rangle = \sqrt{3}M$ for $[A,B] = [1,2] = [6,3] = [4,5]; [\Delta,\Omega] = [4,5]$

$\langle\phi^{[A,B]}_{2[a,b]}\rangle = M$ for $[a,b] = [A,B] = [2,1] = [6,3] = [4,5]$.

It is easy to show[9] that $\langle\phi_1\rangle + \langle\phi_2\rangle$ with the VeVs as indicated breaks

$$G \longrightarrow \mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{SU}(3)_{CL}\otimes\mathrm{SU}(3)_{CR}\otimes\ \mathrm{U}(1)_{(B-L)_L}\otimes\mathrm{U}(1)_{(B-L)_R},$$

the chiral extension of the left-right symmetric extension of the SM.

Next we introduce

$$\phi_3 = \phi_3(5184) = Z_4\phi_3(1,1,(\overline{15}+\overline{21}),(15+21)) = \phi^{ab}_{3,\alpha\eta} + \phi^{\alpha\eta}_{3,\Delta\Omega} + \phi^{\Delta\Omega}_{3,AB} + \phi^{AB}_{3,ab}$$

with the following VEVs:

$\langle\phi^{\alpha\eta}_{3,\Delta\Omega}\rangle = M_C\delta^{\alpha}_{\Omega}\delta^{\eta}_{\Delta}; \qquad \alpha,\eta,\Delta,\Omega = 1,..,6,$

$\langle\phi^{[\Delta,\Omega]}_{3,[A,B]}\rangle = M_R$ for $[\Delta,\Omega] = \Delta\Omega - \Omega\Delta = [A,B] = [4,6],$

$\langle\phi^{ab}_{3,\alpha\eta}\rangle = \langle\phi^{AB}_{3,ab}\rangle = 0.$

It then follows that

$$\mathrm{SU}(6)_{CR}\otimes\mathrm{SU}(6)_{CL} \xrightarrow{\langle\phi^{\alpha\eta}_{3,\Delta\Omega}\rangle} \mathrm{SU}(6)_{(CL+CR)} \equiv\mathrm{SU}(6)^V_C,$$

and that the main effect of $\langle\phi^{[\Delta,\Omega]}_{3,[A,B]}\rangle$ is to break $\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{(B-L)_R}$ in an appropriate way as we will shortly show.

Finally we introduce

$$\phi_4 = \phi_4(2592) = \phi_4(6,\bar{6},6,\bar{6}) + \phi_4(\bar{6},6,\bar{6},6) = \phi^{a\Omega}_{4,A\alpha} + \phi^{A\alpha}_{4,a\Omega}$$

with the following VEVs: $\langle\phi^{A\alpha}_{4,a\Omega}\rangle = 0$, and $\langle\phi^{a\Omega}_{4,A\alpha}\rangle = M_Z$ for (a,A) = (6,6); (Ω,α) = (1,1) = (2,2) = (3,3) = (4,4) = (5,5) = (6,6); and also for (a,A)=(Ω,α)=(5,5). As we will show in the next section the main effect of $\langle\phi_4\rangle$ is to break $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{EM}$.

3. Tree level masses

The scalar fields and their VEVs introduced in the previous section allow for the following tree level masses:

3..1 Masses for gauge bosons

A tedious calculation[9] in the sector of the covariant derivative in the Lagrangian shows the following results:

1. $\langle\phi_1\rangle + \langle\phi_2\rangle$ produces:

$$\begin{aligned}\mathcal{L}(M) &= g^2M^2\Bigg\{18\left[\sum_{\delta=1}^{3}\Big(|X_{\delta CL}|^2 + 2|Y_{\delta CL}|^2 + |Z_{\delta CL}|^2\Big) + 2|P^0_{CL}|^2\right. \\ &+ \left.|P_{1CL}|^2 + |P_{2CL}|^2 + \frac{5}{3}B^2_{1YL} + B^2_{2YL}\right] + 24\left[\sum_{i=1}^{8}c_i|B_{iL}|^2 + \sum_{i=1}^{6}c'_i|H^0_{iL}|^2\right] \\ &+ 12\Big(3A^2_{1HL} + A^2_{2HL} + A^2_{1AL} + 3A^2_{2AL}\Big) + (L \longrightarrow R)\Bigg\},\end{aligned} \tag{7}$$

where g is the gauge coupling constant for the simple group G, and the coefficients c_i and c'_i are such that $c_1 = c_2 = c_3/2 = c_4 = c_5/2 = c_6/3 = c_7 = c_8 = 1$ and $c'_1/3 = c'_2/2 = c'_3 = c'_4/2 = c'_5/3 = c'_6 = 1$. (The relationship between the unprimed fields in Eq. (7) and the primed ones in Eq. (3) is presented in Appendix B.)

As it is clear from the former equation, $\langle\phi_1\rangle + \langle\phi_2\rangle$ breaks G down to the chiral extension of the left-right symmetric extension of the SM.

2. For $\langle\phi_3\rangle$ we split the analysis.

2a. $\langle\phi^{\alpha\eta}_{3,\Delta\Omega}\rangle$ produces:

$$\begin{aligned}\mathcal{L}(M_C) &= 12g^2M_C^2Tr\left[\mathbf{A}^2_{CL} - 2\mathbf{A}_{CL}\mathbf{A}_{CR} + \mathbf{A}^2_{CR}\right] \\ &= 6g^2M_C^2\left[2\sum_{\delta=1}^{3}\Big(|X_{\delta CL} - X_{\delta CR}|^2 + |Y_{\delta CL} - Y_{\delta CR}|^2 + |Z_{\delta CL} - Z_{\delta CR}|^2\Big)\right. \\ &+ \sum_{i=1}^{2}\Big(2|P_{iCL} - P_{iCR}|^2 + (B_{igL} - B_{igR})^2 + (B_{iYL} - B_{iYR})^2\Big)\end{aligned} \tag{8}$$

$$
\begin{aligned}
&+ \quad \Big(B_{(B-L)_L} - B_{(B-L)_R}\Big)^2 + 2|P^0_{CL} - P^0_{CR}|^2 \\
&+ \quad 2\Big(|G^1_{2CL} - G^1_{2CR}|^2 + |G^1_{3CL} - G^1_{3CR}|^2 + |G^2_{3CL} - G^2_{3CR}|^2\Big)\Big].
\end{aligned}
$$

As it is clear from the former expression, $\langle\phi^{\alpha\eta}_{3,\Delta\Omega}\rangle$ breaks $\mathrm{SU}(6)_{CL}\otimes\mathrm{SU}(6)_{CR} \longrightarrow \mathrm{SU}(6)^V_C$ as mentioned before.

2b. $\langle\phi^{[\Delta,\Omega]}_{3,[A,B]}\rangle$ for $[\Delta,\Omega] = [A,B] = [4,6]$ produces:

$$
\begin{aligned}
\mathcal{L}(M_R) \quad &= \quad 2g^2M_R^2\Bigg[\sum_{\delta=1}^{3}\Big(|X_{\delta CR}|^2 + |Z_{\delta CR}|^2\Big) + |P^+_{1CR}|^2 + |P^+_{2CR}|^2 \\
&+ \quad |B_{2R}|^2 + |B_{3R}|^2 + |B'_{6R}|^2 + |B_{7R}|^2 + |B_{8R}|^2 + |B'_{9R}|^2 \\
&+ \quad |H'_{3R}|^2 + |H_{4R}|^2 + \frac{4}{3}\Big(\mathbf{B}^0_{CR} - \mathbf{W}^0_R\Big)^2\Bigg]
\end{aligned} \tag{9}
$$

where

$$
\mathbf{B}^0_{CR} = \Big(3B_{(B-L)_R} + B_{1YR}\Big)/\sqrt{10}
$$

and

$$
\mathbf{W}^0_R = \Big(\sqrt{8}W^0_R + \sqrt{3}A_{1HR} + A_{2HR} - \sqrt{3}A_{1AR} - A_{2AR}\Big)/\sqrt{16}.
$$

The mixing between $\mathrm{SU}(6)_{CR}$ and $\mathrm{SU}(6)_R$ is given by $\mathbf{B}^0_{CR}\cdot\mathbf{W}^0_R$. The analysis shows also that $\langle\phi^{[\Delta,\Omega]}_{3,[A,B]}\rangle$, with the VEVs as stated breaks $\mathrm{SU}(6)_{CR}\otimes\mathrm{SU}(6)_R$ down to $\mathrm{SU}(4)''_{CR}\otimes\mathrm{SU}(2)''_{CR}\otimes\mathrm{SU}(4)''_R\otimes\mathrm{SU}(2)''_R\otimes\mathrm{U}(1)_{mix}$, where $\mathrm{U}(1)_{mix}$ is associated with the unbroken gauge boson $(\mathbf{B}^0_{CR} + \mathbf{W}^0_R)/\sqrt{2}$.

It is also a matter of a careful analysis to realize that $\langle\phi_1\rangle + \langle\phi_2\rangle + \langle\phi_3\rangle$ breaks G down to $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$, the gauge group of the SM.

3. $\langle\phi_4\rangle$ with the VEVs as indicated produces:

$$
\begin{aligned}
\mathcal{L}(M_Z) \quad &= \quad g^2M_Z^2\Big\{Tr\Big[\mathbf{A}^2_{CL} - 2\mathbf{A}_{CL}\mathbf{A}_{CR} + \mathbf{A}^2_{CR} + \mathbf{A}^2_L I^2_5 - 2\mathbf{A}_L I_5 \mathbf{A}_R I_5 + \mathbf{A}^2_R I^2_5 \\
&+ \quad 6\Big(\mathbf{A}^2_L I^2_6 - 2\mathbf{A}_L I_6 \mathbf{A}_R I_6 + \mathbf{A}^2_R I^2_6\Big) + \mathbf{A}^2_{CL} I^2_5 - 2\mathbf{A}_{CL} I_5 \mathbf{A}_{CR} I_5 + \mathbf{A}^2_{CR} I^2_5\Big] \\
&- \quad 2\Big(\mathbf{A}_L - \mathbf{A}_R\Big)^5_5\Big(\mathbf{A}_{CL} - \mathbf{A}_{CR}\Big)^5_5\Big\} \\
&= \quad \frac{M_Z^2}{12M_C^2}\mathcal{L}(M_C) + \mathcal{L}^{LCR}_{\langle\phi_4\rangle},
\end{aligned} \tag{10}
$$

where $\mathcal{L}(M_C)$ is given by Eq. (8), and $I_5 = \delta_{55}$ and $I_6 = \delta_{66}$ are 6×6 matrices with only one entry different from zero, which produce:

$$
\begin{aligned}
\mathcal{L}^{LCR}_{\langle\phi_4\rangle} &= \frac{g^2 M_Z^2}{2}\Bigg[6|B_{3L}|^2 + |B_{5L}|^2 + 6|B_{7L}|^2 + |B_{8L}|^2 + 7|B'_{9L}|^2 + |H^0_{2L}|^2 + 6|H^0_{4L}|^2 \\
&+ |H'^0_{5L}|^2 + 6|H'^0_{6L}|^2 + \left(\sum_{\delta=1}^{3}|Y_{\delta CL}|^2\right) + |P_{1L}|^2 + |P_{2L}|^2 + (L \longrightarrow R) \\
&+ 6(A_{6L} - A_{6R})^2 + (A_{5L} - D_{5L} - A_{5R} + D_{5R})^2\Bigg] \qquad (11)
\end{aligned}
$$

Combining the former equations we see that the only gauge bosons that remain massless are:

1. The eight fields $G^{V,\delta}_{\eta} = (G^{\delta}_{\eta,CL} + G^{\delta}_{\eta,CR})/\sqrt{2}, \delta, \eta = 1,2,3, (\sum_{\delta} G^{V,\delta}_{\delta} = 0)$, associated with the gauge bosons for SU$(3)_C$.

2. $A = \frac{3}{\sqrt{28}}[W^0_L + W^0_R - \frac{\sqrt{5}}{3}(B_{(B-L)_L} + B_{(B-L)_R})]$ which is the photon field. Then using the identity $A = sin\theta_W W^0_L + cos\theta_W B_Y$, where θ_W is the weak mixing angle, we get $sin\theta_W = 3/\sqrt{28}$ at the G scale, and $B_Y = [3W^0_R - \sqrt{5}(B_{(B-L)_L} + B_{(B-L)_R})]/\sqrt{19}$ as the boson associated with U$(1)_Y$.

3..2 Masses for fermion fields

With the scalar fields of the model $\phi_i, i = 1, ..., 4$ we can construct the following Yukawa terms:

$$
Z_4\psi(\bar{6},1,1,6)\psi(\bar{6},1,1,6)[\sum_{i=1}^{2} y_i\phi_i(15,1,1,\overline{15}) + y_3\phi_3((15+21),1,1,(\overline{15}+\overline{21}))]
$$

$$
+y_4 Z_4\psi(\bar{6},1,1,6)\psi(1,6,\bar{6},1)\phi_4(6,\bar{6},6,\bar{6}) + h.c.
$$

where $y_i, i = 1, ...4$ are Yukawa coupling constants of order one. When the Higgs fields $\phi_i, i = 1-4$ develop the VEVs as indicated in Section 2.3 they produce the following masses for the fermion fields:

3..2.1 MASSES FROM $\langle\phi_1\rangle + \langle\phi_2\rangle$

$Z_4\psi(\bar{6},1,1,6)\psi(\bar{6},1,1,6)\sum_{i=1}^{2} y_i\langle\phi_i(15,1,1,\overline{15})\rangle$ produces

1. Masses of order M for all the exotic *nones* in $\psi(6,\bar{6},1,1)$.

2. The following Dirac masses:

$$
\begin{aligned}
\mathcal{L}_M^{\langle\phi_1\rangle+\langle\phi_2\rangle} &= n_{12}^{+}(Y_1e_{23}^{-}+Y_2e_{11}^{-})+n_{22}^{+}(Y_1e_{13}^{-}+Y_2e_{31}^{-})+n_{32}^{+}(Y_1e_{33}^{-}+Y_2e_{21}^{-})\\
&+ N_{12}^{-}(Y_1E_{23}^{+}+Y_2E_{11}^{+})+N_{22}^{-}(Y_1E_{13}^{+}+Y_2E_{31}^{+})+N_{32}^{-}(Y_1E_{33}^{+}+Y_2E_{21}^{+})\\
&+ E_{12}^{0}(Y_1N_{23}^{0c}+Y_2N_{11}^{0c})+E_{22}^{0}(Y_1N_{13}^{0c}+Y_2N_{31}^{0c})+E_{32}^{0}(Y_1N_{33}^{0c}+Y_2N_{21}^{0c})\\
&+ e_{12}^{0c}(Y_1n_{23}^{0}+Y_2n_{11}^{0})+e_{22}^{0c}(Y_1n_{13}^{0}+Y_2n_{31}^{0})+e_{32}^{0c}(Y_1n_{33}^{0}+Y_2n_{21}^{0})\\
&+ h.c., \qquad (12)
\end{aligned}
$$

where $Y_i=\sqrt{3}My_i$, $i=1,2$. Equation (12) allows us to identify $\kappa(Y_2e_{23}-Y_1e_{11})$, $\kappa(Y_2e_{13}-Y_1e_{31})$ and $\kappa(Y_2e_{33}^{-}-Y_1e_{21}^{-})$ with $\kappa=(Y_1^2+Y_2^2)^{-1/2}$ as a basis for the known charged left-handed leptons $(e^-,\mu^-,\tau^-)_L$; $\kappa(Y_2E_{23}^{+}-Y_1E_{11}^{-})$, $\kappa(Y_2E_{13}^{+}-Y_1E_{31}^{+})$ and $\kappa(Y_2E_{33}^{+}-Y_1E_{21}^{-})$ as a basis for the known charged right-handed leptons $(e^-,\mu^-,\tau^-)_R$; $\kappa(Y_2n_{23}^{0}-Y_1n_{11}^{0})$, $\kappa(Y_2n_{31}^{0}-Y_1n_{13}^{0})$ and $\kappa(Y_2n_{33}^{0}-Y_1n_{21}^{0})$ as a basis for $(\nu_e,\nu_\mu,\nu_\tau)_L$, and $\kappa(Y_2N_{23}^{0c}-Y_1N_{11}^{0c})$, $\kappa(Y_2N_{31}^{0c}-Y_1N_{13}^{0c})$ and $\kappa(Y_2N_{33}^{0c}-Y_1N_{21}^{0c})$ as a basis for $(\nu_e^c,\nu_\mu^c,\nu_\tau^c)_L$.

As can be seen from the former expression, all the vector-like particles with respect to the chiral extension of the left-right symmetric extension of the SM acquire masses of order M, as it should be according to the survival hypothesis[2] (see Appendix A).

3..2.2 MASSES FROM $\langle\phi_3\rangle$

$Z_4\psi(1,1,6,\bar{6})\psi(1,1,6,\bar{6})\langle\phi_3(1,1,(\overline{15}+\overline{21}),(15+21))\rangle$ producces the following masses:

1. Dirac masses for all the exotic fields in $\psi(1,1,6,\bar{6})$ of order M_C, via the Yukawa term $y_3\psi_\alpha^\Delta\psi_\eta^\Omega\langle\phi_{3,\Delta\Omega}^{\alpha\eta}\rangle$

2. The following Majorana masses:

$$\begin{aligned}\mathcal{L}_{M_R}^{\langle\phi_3\rangle} &= y_3\psi_\Delta^A\psi_\Omega^B\langle\phi_{3,[A,B]}^{[\Delta,\Omega]}\rangle \\ &= y_3M_R(N_{21L}^{0c}N_{33L}^{0c}+N_{23L}^{0c}N_{31L}^{0c}+N_{33L}^{0c}N_{21L}^{0c}+N_{31L}^{0c}N_{23L}^{0c}),\end{aligned} \tag{13}$$

3..2.3 Masses from $\langle\phi_4\rangle$

ϕ_4, with the VEVs as stated in the previous section, produces the following mass terms:

$$\begin{aligned}\mathcal{L}_{M_Z}^{\langle\phi_4\rangle} &= y_4\left(\psi_a^\alpha\psi_\Delta^A+\psi_\Delta^A\psi_a^\alpha\right)\langle\phi_{4,\alpha A}^{a\Delta}\rangle \qquad (14)\\ &= y_4M_Z\left[\sum_{\delta=1}^{3}t_{\delta L}^c t_{\delta L}+N_{31L}^{0c}n_{31L}^0+N_{32L}^-n_{32L}^+ +N_{33L}^{0c}n_{33L}^0+E_{32L}^0e_{32L}^{0c}+h.c.\right],\end{aligned}$$

from where we can immediately see that the top quark (but not the bottom quark) gets a tree level mass $m_t = y_4M_Z$. The algebra also shows that Eq.[14] contains a small mass term for one of the neutrino fields[10]. This is the way how we achieve the modified horizontal survival hypothesis in the context of the model presented here.

4. Mass scales

4..1 The electroweak mixing angle

There are several ways to calculate the electroweak mixing angle at the unification scale (M_G) for a grand unified theory. For a simple gauge group the relationship[11]

$$sin^2\theta_W(M_G) = tr(T_{ZL}^2)/tr(Q^2),$$

may be used, where the traces can be evaluated using any faithful representation (reducible or irreducible) of the simple group.

Now, $[\mathrm{SU}(6)]^4$ is not simple, but $[\mathrm{SU}(6)]^4\times Z_4$ is. Therefore we can calculate the traces for $\psi(144)$ and plug them in the former expression. Note that all the four sectors of

$\psi(144)$ must be used in the computation of the traces due to the fact that a single sector is not a faithful representation of G because it is not Z_4 invariant. After the algebra is done we get $sin^2\theta_W(M_G) = 9/28$ in agreement with the previous calculation, and the same value obtained for the three family extension of the Pati-Salam model with mirror fermions[12].

Now, if we define g_1, g_2, and g_3 as the gauge coupling constants for U$(1)_Y$, SU$(2)_L$, and SU$(3)_C$ respectively, the the embedding of the SM model gauge group for three families in $[G, \psi(144)]$, and the former value for $sin^2\theta_W$ imply that at the G scale the following relationships holds[3, 12]: $g_3 = g/\sqrt{2}, g_2 = g/\sqrt{3}$, and $g_1 = \sqrt{3/19}g$. At scales well below the G scale the former relations are not longer valid because the embedding symmetry G is not manifest, then the effective coupling constants must be evaluated using the renormalization group equations.

4..2 The renormalization group equations

Next we introduce the renormalization group equations and use standard decoupling theorem arguments[13] in order to calculate the mass scales.

For generality, let us analyze the two mass scale symmetry breaking pattern

$$G \stackrel{M_R=M_C}{\longrightarrow} G_I \stackrel{M}{\longrightarrow} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \stackrel{M_Z}{\longrightarrow} SU(3)_C \otimes U(1)_{EM}$$

with $M_R >> M >> M_Z$, and G_I = SU$(6)_L \otimes$ SU$(4)^V_C \otimes$ U$(1)_Y$ $\otimes$ ..., where SU$(3)_C$ $\subset$ SU$(4)^V_C$ and SU$(2)_L$ $\subset$ SU$(6)_L$. For this two-stage gauge hierarchy the runing coupling constants of the SM satisfy the one loop renormalization group equations[14]

$$\alpha_i^{-1}(M_Z) = f_i\alpha^{-1} - b_i^{M_R}\ln\left(\frac{M_R}{M}\right) - b_i^{M}\ln\left(\frac{M}{M_Z}\right), \tag{15}$$

where $\alpha_i = g_i^2/4\pi,\ i = 1, 2, 3, \alpha = g^2/4\pi$, and f_i are embedding constants given by

$f_1 = 19/3, f_2 = 3$ and $f_3 = 2$. The beta functions are:

$$b_i = \{\frac{11}{3}C_i(vectors) - \frac{2}{3}C_i(Weyl - fermions) - \frac{1}{6}C_i(scalars)\}/4\pi, \tag{16}$$

where $C_i(...)$ is the index of the representation to which the (...) particles are assigned, and the $C_i(Weyl - fermions)$ and $C_i(scalars)$ indexs must be properly normalized with the embedding factor f_i.

Now, using the relationships $e^{-2} = g_1^{-2} + g_2^{-2}$ and $\tan\theta_W = g_1/g_2$, valid at all energy scales, we get from Eqs(15):

$$\frac{3}{28}\alpha_{EM}^{-1}(M_Z) = \frac{\alpha_3^{-1}(M_Z)}{2} + (\frac{b_3^{M_R}}{2} - \frac{3b_2^{M_R}}{28} - \frac{3b_1^{M_R}}{28})\ln\left(\frac{M_R}{M}\right) + (\frac{b_3^{M}}{2} - \frac{3b_2^{M}}{28} - \frac{3b_1^{M}}{28})\ln\left(\frac{M}{M_Z}\right), \tag{17}$$

and

$$\sin^2\theta_W(M_Z) = 3\alpha_{EM}(M_Z)\{\frac{\alpha_3^{-1}(M_Z)}{2} + (\frac{b_3^{M_R}}{2} - \frac{b_2^{M_R}}{3})\ln\left(\frac{M_R}{M}\right) + (\frac{b_3^{M}}{2} - \frac{b_2^{M}}{3})\ln\left(\frac{M}{M_Z}\right)\}. \tag{18}$$

As it is well known, the Higgs fields play an important role in the beta functions[15] and can drastically change the solutions to the renormalization group equations. So, we are going to solve those equations under the assumption that the extended survival hypothesis holds[15]. Using this hypothesis, decoupling the vector-like representations in $\psi(144)$ according to the Appelquist–Carazzone theorem[13], and using the experimental values[16] $sin^2\theta_W(M_Z) = 0.2319, \alpha_3(M_Z) = 0.117$ and $\alpha_{EM}^{-1}(M_Z) = 127.6$ we get the solutions $M = 5.0 \times 10^5 M_Z$ and $M_R = 5.5M$. When the threshold effects and the experimental errors are taken into account, the solution is compatible with the amazing result $M_R = M_C \sim M \sim 10^8 >> M_Z \sim 10_2$ GeVs, which implies that only one stage symmetry breaking pattern is required, and there is only one mass scale between the G and the electroweak scales.

So, our model is compatible with the symmetry breaking pattern:

$$G \xrightarrow{M} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{EM},$$

where $M \sim 10^8$ GeVs, and $M_Z \sim 10^2$GeVs is the electroweak mass scale. Notice also that the lower value of the G scale softens the gauge hierarchy problem.

5. Interacting Lagrangian

Using the covariant derivative for G we can write the following interacting terms:

$$
\begin{aligned}
\mathcal{L}^{int} &= g[\psi(\bar{6},1,1,6)\mathbf{A}_{CL}\psi(\bar{6},1,1,6) - \psi(\bar{6},1,1,6)\mathbf{A}_{L}\psi(\bar{6},1,1,6) \\
&+ \psi(1,6,\bar{6},1)\mathbf{A}_{R}\psi(1,6,\bar{6},1) - \psi(1,6,\bar{6},1)\mathbf{A}_{CR}\psi(1,6,\bar{6},1) \\
&+ \psi(1,1,6,\bar{6})\mathbf{A}_{CR}\psi(1,1,6,\bar{6}) - \psi(1,1,6,\bar{6})\mathbf{A}_{CL}\psi(1,1,6,\bar{6}) \\
&+ \psi(6,\bar{6},1,1)\mathbf{A}_{L}\psi(6,\bar{6},1,1) - \psi(6,\bar{6},1,1)\mathbf{A}_{R}\psi(6,\bar{6},1,1)] \\
&\equiv \mathcal{L}_{CL} + \mathcal{L}_{R} - \mathcal{L}_{L} - \mathcal{L}_{CR}.
\end{aligned}
\tag{19}
$$

As far as the ordinary particles are concerned, each term in $\mathcal{L}^{int}$ may be written as

$$\mathcal{L}_i = \mathcal{L}_i^{qq} + \mathcal{L}_i^{ql} + \mathcal{L}_i^{ll}$$

for $i = CL, R, L, CL$, where qq, ql, and ll stand for quark-quark, quark-lepton and lepton-lepton interactions respectively. Also for our concern here, only the terms in Eq. (19) with known fields must be evaluated explicitly.

After the algebra is done we get the following expressions:

$$\mathcal{L}_{CL}^{qq} = \frac{g}{\sqrt{2}}\left\{\sum_{q=u,d,c,s,t,b}\left[\sum_{\delta\neq\eta=1}^{3}\bar{q}_{\delta L}(G_\eta^\delta)_{CL}^\mu\gamma_\mu q_{\eta L} + \sum_{\delta=1}^{3}\bar{q}_{\delta L}D_{\delta CL}^\mu\gamma_\mu q_{\delta L}\right]\right\}, \tag{20}$$

$$
\begin{aligned}
\mathcal{L}_{CL}^{ql} &= \frac{g}{\sqrt{2}}\sum_{\delta=1}^{3}\Big[X_{\delta CL}^\mu(\bar{\mathbf{n}}_{1L}^0\cdot\gamma_\mu U_{\delta L} + \bar{\mathbf{e}}_{1L}^-\cdot\gamma_\mu D_{\delta L}) + Y_{\delta CL}^\mu(\bar{\mathbf{n}}_{2L}^+\cdot\gamma_\mu U_{\delta L} + \bar{\mathbf{e}}_{2L}^{0c}\cdot\gamma_\mu D_{\delta L}) \\
&+ Z_{\delta CL}^\mu(\bar{\mathbf{n}}_{3L}^0\cdot\gamma_\mu U_{\delta L} + \bar{\mathbf{e}}_{3L}^-\cdot\gamma_\mu D_{\delta L}) + h.c.\Big],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathcal{L}^{ll}_{CL} &= \frac{g}{\sqrt{2}}\Big[D^{\mu}_{4CL}(\bar{\mathbf{n}}^0_{1L}\cdot\gamma_\mu\mathbf{n}^0_{1L}+\bar{\mathbf{e}}^-_{1L}\cdot\gamma_\mu\mathbf{e}^-_{1L})+D^{\mu}_{5CL}(\bar{\mathbf{n}}^+_{2L}\cdot\gamma_\mu\mathbf{n}^+_{2L}+\bar{\mathbf{e}}^{0c}_{2L}\cdot\gamma_\mu\mathbf{e}^{0c}_{2L})\\
&+ D^{\mu}_{6CL}(\bar{\mathbf{n}}^0_{3L}\cdot\gamma_\mu\mathbf{n}^0_{3L}+\bar{\mathbf{e}}^-_{3L}\cdot\gamma_\mu\mathbf{e}^-_{3L})+P^{0,\mu}_{CL}(\bar{\mathbf{n}}^0_{1L}\cdot\gamma_\mu\mathbf{n}^0_{3L}+\bar{\mathbf{e}}^-_{1L}\cdot\gamma_\mu\mathbf{e}^-_{3L})\\
&+ P^{+,\mu}_{1CL}(\bar{\mathbf{n}}^+_{2L}\cdot\gamma_\mu\mathbf{n}^0_{1L}+\bar{\mathbf{e}}^{0c}_{2L}\cdot\gamma_\mu\mathbf{e}^-_{1L})+P^{+,\mu}_{2CL}(\bar{\mathbf{n}}^+_{2L}\cdot\gamma_\mu\mathbf{n}^0_{3L}+\bar{\mathbf{e}}^{0c}_{2L}\cdot\gamma_\mu\mathbf{e}^-_{3L})\\
&+ h.c.\Big], \qquad (22)
\end{aligned}
$$

where we have defined the following three component vectors: $U_\delta = (u_\delta, c_\delta, t_\delta)$, $D_\delta = (d_\delta, s_\delta, b_\delta)$; $\mathbf{n}^0_1 = (-n^0_{11}, n^0_{21}, n^0_{31})$, $\mathbf{n}^+_2 = (n^+_{12}, n^+_{22}, n^+_{32})$, $\mathbf{n}^0_3 = (-n^0_{13}, -n^0_{23}, n^0_{33})$; $\mathbf{e}^-_1 = (e^-_{11}, -e^-_{21}, -e^-_{31})$, $\mathbf{e}^{0c}_2 = (e^{0c}_{12}, e^{0c}_{22}, e^{0c}_{32})$ and $\mathbf{e}^-_3 = (e^-_{13}, e^-_{23}, -e^-_{33})$.

$\mathcal{L}^{qq}_{CR}, \mathcal{L}^{ql}_{CR}$ and $\mathcal{L}^{ll}_{CR}$ are expressions similar to the ones pressented in Eqs. (20), (21) and (22) with the following changes: replacement of $CL \to CR$ in all the gauge fields, changing of the quark fields U_L and D_L by their corresponding charge conjugated fields U^c_L and D^c_L, and changing of the lepton vectors $\mathbf{n}^0_1$, $\mathbf{n}^+_2$, $\mathbf{n}^0_3$, $\mathbf{e}^-_1$, $\mathbf{e}^{0c}_2$ and $\mathbf{e}^-_3$ by $\mathbf{N}^{0c}_1 = (-N^{0c}_{11}, N^{0c}_{21}, N^{0c}_{31})$, $\mathbf{N}^-_2 = (N^-_{12}, N^-_{22}, N^+_{32})$, $\mathbf{N}^{0c}_3 = (-N^{0c}_{13}, -N^{0c}_{23}, N^{0c}_{33})$; $\mathbf{E}^+_1 = (E^+_{11}, -E^+_{21}, -E^+_{31})$, $\mathbf{E}^0_2 = (E^0_{12}, E^0_{22}, E^0_{32})$ and $\mathbf{E}^+_3 = (E^+_{13}, E^+_{23}, -E^+_{33})$, respectively. Then the right-handed fields will show up in the final expressions by using the identity $\bar{\chi}^c_L\gamma^\mu\xi^c_L = -\bar{\xi}_R\gamma^\mu\chi_R$. Then $-\mathcal{L}_{CR}$ will be just $\mathcal{L}_{CL}$ with the substitutions $L \to R$ and $\{\mathbf{n}, \mathbf{e}\} \to \{\mathbf{N}, \mathbf{E}\}$ everywhere.

Next for $\mathcal{L}_L$ and $\mathcal{L}_R$ we have the result that $\mathcal{L}^{ql}_{L(R)} = 0$. Then $\mathcal{L}^{qq}_{L(R)}$ and $\mathcal{L}^{ll}_{L(R)}$ can be conveniently written as:

$$
\mathcal{L}^{qq(ll)}_{L(R)} = \mathcal{L}^{q(l)W}_{L(R)} + \mathcal{L}^{q(l)H}_{L(R)} + \mathcal{L}^{q(l)A}_{L(R)} + \mathcal{L}^{q(l)B}_{L(R)}.
$$

After the algebra is done we get the following expressions

$$
\mathcal{L}^{qW}_L = \frac{g}{\sqrt{6}}\sum_{\delta=1}^{3}\left[\frac{W^0_{\mu L}}{\sqrt{2}}\Big(\bar{D}_{\delta L}\cdot\gamma^\mu D_{\delta L} - \bar{U}_{\delta L}\cdot\gamma^\mu U_{\delta L}\Big) + \Big(W^+_{\mu L}\bar{U}_{\delta L}\cdot\gamma^\mu D_{\delta L} + h.c.\Big)\right], \qquad (23)
$$

$$
\mathcal{L}^{qH}_L = \frac{g}{\sqrt{2}}\sum_{\delta=1}^{3}\Big[\frac{H^{0\mu}_{1L}}{\sqrt{2}}(\bar{c}_{\delta L}\gamma_\mu u_{\delta L} + \bar{s}_{\delta L}\gamma_\mu d_{\delta L}) + H^{0\mu}_{2L}\,\bar{b}_{\delta L}\gamma_\mu d_{\delta L}
$$

$$
\begin{aligned}
&+ \frac{H^{0\mu}_{3L}}{\sqrt{2}}(\bar{c}_{\delta L}\gamma_\mu u_{\delta L} - \bar{s}_{\delta L}\gamma_\mu d_{\delta L}) + H^{0\mu}_{4L}\ \bar{t}_{\delta L}\gamma_\mu u_{\delta L} \\
&+ \frac{H^{0\mu}_{5L}}{\sqrt{2}}(\bar{b}_{\delta L}\gamma_\mu s_{\delta L} + \bar{t}_{\delta L}\gamma_\mu c_{\delta L}) + \frac{H^{0\mu}_{6L}}{\sqrt{2}}\left(\bar{t}_{\delta L}\gamma_\mu c_{\delta L} - \bar{b}_{\delta L}\gamma_\mu s_{\delta L}\right) + h.c.\Big],
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\mathcal{L}^{qA}_L &= \frac{g}{2\sqrt{2}}\sum_{\delta=1}^{3}\Big[A^\mu_{1HL}(\bar{d}_{\delta L}\gamma_\mu d_{\delta L} + \bar{u}_{\delta L}\gamma_\mu u_{\delta L} - \bar{b}_{\delta L}\gamma_\mu b_{\delta L} - \bar{t}_{\delta L}\gamma_\mu t_{\delta L}) \\
&+ \frac{A^\mu_{2HL}}{\sqrt{3}}(\bar{d}_{\delta L}\gamma_\mu d_{\delta L} + \bar{u}_{\delta L}\gamma_\mu u_{\delta L} - 2\bar{s}_{\delta L}\gamma_\mu s_{\delta L} - 2\bar{c}_{\delta L}\gamma_\mu c_{\delta L} + \bar{b}_{\delta L}\gamma_\mu b_{\delta L} + \bar{t}_{\delta L}\gamma_\mu t_{\delta L}) \\
&+ \frac{A^\mu_{2AL}}{\sqrt{3}}(\bar{d}_{\delta L}\gamma_\mu d_{\delta L} - \bar{u}_{\delta L}\gamma_\mu u_{\delta L} - 2\bar{s}_{\delta L}\gamma_\mu s_{\delta L} + 2\bar{c}_{\delta L}\gamma_\mu c_{\delta L} + \bar{b}_{\delta L}\gamma_\mu b_{\delta L} - \bar{t}_{\delta L}\gamma_\mu t_{\delta L}) \\
&+ A^\mu_{1AL}(\bar{d}_{\delta L}\gamma_\mu d_{\delta L} - \bar{u}_{\delta L}\gamma_\mu u_{\delta L} - \bar{b}_{\delta L}\gamma_\mu b_{\delta L} + \bar{t}_{\delta L}\gamma_\mu t_{\delta L})\Big],
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
\mathcal{L}^{qB}_L &= \frac{g}{\sqrt{2}}\sum_{\delta=1}^{3}\Big[\frac{B^{-\mu}_{1L}}{\sqrt{2}}(\bar{d}_{\delta L}\gamma_\mu u_{\delta L} - \bar{b}_{\delta L}\gamma_\mu t_{\delta L}) + B^{-\mu}_{2L}\ \bar{d}_{\delta L}\gamma_\mu c_{\delta L} + B^{-\mu}_{3L}\ \bar{d}_{\delta L}\gamma_\mu t_{\delta L} \\
&+ B^{-\mu}_{4L}\ \bar{s}_{\delta L}\gamma_\mu u_{\delta L} + B^{-\mu}_{5L}\ \bar{b}_{\delta L}\gamma_\mu u_{\delta L} + \frac{B^{-\mu}_{6L}}{\sqrt{6}}(\bar{b}_{\delta L}\gamma_\mu t_{\delta L} - 2\bar{s}_{\delta L}\gamma_\mu c_{\delta L} + \bar{d}_{\delta L}\gamma_\mu u_{\delta L}) \\
&+ B^{-\mu}_{7L}\ \bar{s}_{\delta L}\gamma_\mu t_{\delta L} + B^{-\mu}_{8L}\ \bar{b}_{\delta L}\gamma_\mu c_{\delta L} + h.c.\Big].
\end{aligned} \tag{26}
$$

Again, $-\mathcal{L}^{qq}_R$ is just $\mathcal{L}^{qq}_L$ with the substitution $L \to R$ everywhere.

The expressions for $\mathcal{L}^{ll}_{L(R)}$ are very similar to $\mathcal{L}^{qq}_{L(R)}$. In fact $\mathcal{L}^{ll}_L$ is just $\mathcal{L}^{qq}_L$ with the substitutions $D_\delta \to \mathbf{e}_\delta$ and $U_\delta \to \mathbf{n}_\delta$, in the expression for $\mathcal{L}^{qW}_L$, and $u_\delta \to \eta_1 = (-n^0_{11}, n^+_{12}, -n^0_{13})$, $c_\delta \to \eta_2 = (n^0_{21}, n^+_{22}, -n^0_{23})$, $t_\delta \to \eta_3 = (n^0_{31}, n^+_{32}, n^0_{33})$, $d_\delta \to \varepsilon_1 = (e^-_{11}, e^{0c}_{12}, e^-_{13})$, $s_\delta \to \varepsilon_2 = (-e^-_{21}, e^{0c}_{22}, e^-_{23})$ and $b_\delta \to \varepsilon_3 = (-e^-_{31}, e^{0c}_{32}, -e^-_{33})$; and the exclusion of the sum in the other expressions. Now, $-\mathcal{L}^{ll}_R$ is just $\mathcal{L}^{ll}_L$ with the substitutions $L \to R$ and $\{e_{ij}, n_{ij}\} \to \{E_{ij}, N_{ij}\}$ everywhere.

If now one introduces instead of the mathematical leptons introduced in $\psi(\bar{6},1,1,6) \oplus \psi(1,6,\bar{6},1)$, the more natural set of lepton fields $l = (e,\mu,\tau)$, $\nu = (\nu_e,\nu_\mu,\nu_\tau)$, $\mathbf{n} = (n_{12}, n_{22}, n_{32})$ and $\mathbf{e} = (e^0_{12}, e^0_{22}, e^0_{32})$, given by

$$
(e^-_{23}, e^-_{11})_L = (n^-_{12}, e^-)_L\mathcal{M}; \quad (e^-_{13}, e^-_{31})_L = (n^-_{22}, \mu^-)_L\mathcal{M}; \quad (e^-_{33}, e^-_{21})_L = (n^-_{32}, \tau^-)_L\mathcal{M}
$$

$$(E^+_{23},E^+_{11})_L=(N^+_{12},e^+)_L\mathcal{M};\quad (E^+_{13},E^+_{31})_L=(N^+_{22},\mu^+)_L\mathcal{M};\quad (E^+_{33},E^+_{21})_L=(N^+_{32},\tau^+)_L\mathcal{M}$$

$$(n^0_{23},n^0_{11})_L=(e^0_{12},\nu_e)_L\mathcal{M};\quad (n^0_{13},n^0_{31})_L=(e^0_{22},\nu_\mu)_L\mathcal{M};\quad (n^0_{33},n^0_{21})_L=(e^0_{32},\nu_\tau)_L\mathcal{M}$$

$$(N^{0c}_{23},N^{0c}_{11})_L=(E^{0c}_{12},\nu^c_e)_L\mathcal{M};\quad (N^{0c}_{13},N^{0c}_{31})_L=(E^{0c}_{22},\nu^c_\mu)_L\mathcal{M};\quad (N^{0c}_{33},N^{0c}_{21})_L=(E^{0c}_{32},\nu^c_\tau)_L\mathcal{M},$$

where

$$\mathcal{M}=\kappa\begin{pmatrix} Y_1 & Y_2 \\ Y_2 & -Y_1\end{pmatrix},$$

then the former Lagrangians can be put into the form

$$\begin{aligned}
\mathcal{L}^{ql}_{CL} &= \frac{g}{\sqrt{2}}\sum_{\delta=1}^{3}\Big[\kappa X^\mu_{\delta CL}\Big(Y_1(\bar\nu_e,-\bar\nu_\tau,-\bar\nu_\mu)_L\cdot\gamma_\mu U_{\delta L}+Y_1(-\bar e^-,\bar\tau^-,\bar\mu^-)_L\cdot\gamma_\mu D_{\delta L}\\
&+ Y_2(-\bar e^0_{12},\bar e^0_{32},\bar e^0_{22})_L\cdot\gamma_\mu U_{\delta L}+Y_2(\bar n^-_{12},-\bar n^-_{32},-\bar n^-_{22})_L\cdot\gamma_\mu D_{\delta L}\Big)\\
&+ Y^\mu_{\delta CL}(\bar{\mathbf{n}}^+_L\cdot\gamma_\mu U_{\delta L}+\bar{\mathbf{e}}^c_L\cdot\gamma_\mu D_{\delta L})\\
&+ \kappa Z^\mu_{\delta CL}\Big(Y_1(-\bar e^0_{22},-\bar e^0_{12},\bar e^0_{32})_L\cdot\gamma_\mu U_{\delta L}+Y_1(\bar n^-_{22},\bar n^-_{12},-\bar n^-_{32})_L\cdot\gamma_\mu D_{\delta L}\\
&+ Y_2(-\bar\nu_\mu,-\bar\nu_e,\bar\nu_\tau)_L\cdot\gamma_\mu U_{\delta L}+Y_2(\bar\mu^-,\bar e^-,-\bar\tau^-)_L\cdot\gamma_\mu D_{\delta L}\Big)+h.c.\Big],
\end{aligned}\tag{27}$$

$$\begin{aligned}
\mathcal{L}^{ll}_{CL} &= \frac{g}{\sqrt{2}}\Big\{D^\mu_{4CL}\,\kappa^2\Big[Y_1^2(\bar l^-_L\cdot\gamma_\mu l^-_L+\bar\nu_L\cdot\gamma_\mu\nu_L)+Y_2^2(\bar{\mathbf{n}}^-_L\cdot\gamma_\mu\mathbf{n}^-_L+\bar{\mathbf{e}}_L\cdot\gamma_\mu\mathbf{e}_L)\\
&- Y_1Y_2(\bar{\mathbf{n}}^-_L\cdot\gamma_\mu l^-_L+\bar{\mathbf{e}}_L\cdot\gamma_\mu\nu_L+h.c)\Big]+D^\mu_{5CL}\Big[\bar{\mathbf{n}}^+_L\cdot\gamma_\mu\mathbf{n}^+_L+\bar{\mathbf{e}}^c_L\cdot\gamma_\mu\mathbf{e}^c_L\Big]\\
&+ D^\mu_{6CL}\,\kappa^2\Big[Y_1^2(\bar{\mathbf{n}}^-_L\cdot\gamma_\mu\mathbf{n}^-_L+\bar{\mathbf{e}}_L\cdot\gamma_\mu\mathbf{e}_L)+Y_2^2(\bar l^-_L\cdot\gamma_\mu l^-_L+\bar\nu_L\cdot\gamma_\mu\nu_L)\\
&+ Y_1Y_2(\bar{\mathbf{n}}^-_L\cdot\gamma_\mu l_L+\bar{\mathbf{e}}_L\cdot\gamma_\mu\nu_L+h.c)\Big]+P^{0,\alpha}_{CL}\,\kappa^2\Big[Y_1^2(\bar l_{\tau L}\cdot\gamma_\alpha l_{1L}-\bar l_{eL}\cdot\gamma_\alpha l_{2L}-\bar l_{\mu L}\cdot\gamma_\alpha l_{3L})\\
&+ Y_1Y_2(\bar l_{1L}\cdot\gamma_\alpha l_{2L}+\bar l_{2L}\cdot\gamma_\alpha l_{3L}-\bar l_{3L}\cdot\gamma_\alpha l_{1L}+\bar l_{\tau L}\cdot\gamma_\alpha l_{eL}-\bar l_{eL}\cdot\gamma_\alpha l_{\mu L}-\bar l_{\mu L}\cdot\gamma_\alpha l_{\tau L})\\
&+ Y_2^2(\bar l_{1L}\cdot\gamma_\alpha l_{\mu L}+\bar l_{2L}\cdot\gamma_\alpha l_{\tau L}+\bar l_{3L}\cdot\gamma_\alpha l_{eL})\Big]+P^{+,\alpha}_{1CL}\,\kappa\Big[Y_1(\bar l^c_{3L}\sigma_2\cdot\gamma_\alpha l_{\mu L}+\bar l^c_{2L}\sigma_2\cdot\gamma_\alpha l_{\tau L}\\
&- \bar l^c_{1L}\sigma_2\cdot\gamma_\alpha l_{eL})+Y_2(\bar n^+_{32}\gamma_\alpha e^0_{22}+\bar n^+_{22}\gamma_\alpha e^0_{32}-\bar n^+_{12}\gamma_\alpha e^0_{12})\Big]\\
&+ P^{+,\alpha}_{2CL}\,\kappa\Big[Y_1(\bar n^+_{32}\gamma_\alpha e^0_{32}-\bar n^+_{22}\gamma_\alpha e^0_{12}-\bar n^+_{12}\gamma_\alpha e^0_{22})+Y_2(\bar l^c_{1L}\sigma_2\cdot\gamma_\alpha l_{\mu L}+\bar l^c_{2L}\sigma_2\cdot\gamma_\alpha l_{eL}\\
&- \bar l^c_{3L}\sigma_2\cdot\gamma_\alpha l_{\tau L})\Big]+h.c.\Big\},
\end{aligned}\tag{28}$$

$$
\begin{aligned}
\mathcal{L}_L^{lW} &= \frac{g}{\sqrt{6}}\Big\{\frac{W_{\mu L}^0}{\sqrt{2}}\Big[\bar{\mathbf{n}}_L^- \cdot\gamma^\mu \mathbf{n}_L^- - \bar{\mathbf{e}}_L\cdot\gamma^\mu \mathbf{e}_L + \bar{l}_L^-\cdot\gamma_\mu l_L^- - \bar{\nu}_L\cdot\gamma_\mu\nu_L\Big]\\
&- \Big[W_{\mu L}^+(\bar{\nu}_L\cdot\gamma_\mu l_L^- + \bar{\mathbf{e}}_L\cdot\gamma^\mu\mathbf{n}_L^-) + h.c\Big]\Big\},
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\mathcal{L}_L^{lH} &= \frac{g}{\sqrt{2}}\Big\{\frac{H_{1L}^{0\alpha}}{\sqrt{2}}\Big(\bar{l}_{2L}\cdot\gamma_\alpha l_{1L} + \kappa^2 Y_1^2(\bar{l}_{1L}\cdot\gamma_\alpha l_{2L} - \bar{l}_{\tau L}\cdot\gamma_\alpha l_{eL}) + \kappa^2 Y_2^2(\bar{l}_{eL}\cdot\gamma_\alpha l_{\mu L}\\
&- \bar{l}_{3L}\cdot\gamma_\alpha l_{1L}) + \kappa^2 Y_1 Y_2(\bar{l}_{3L}\cdot\gamma_\alpha l_{eL} + \bar{l}_{1L}\cdot\gamma_\alpha l_{\mu L} + \bar{l}_{eL}\cdot\gamma_\alpha l_{2L} + \bar{l}_{\tau L}\cdot\gamma_\alpha l_{1L})\Big]\\
&+ H_{2L}^{0\alpha}\Big[\bar{e}_{32L}^{0c}\gamma_\alpha E_{32L}^{0c} - \kappa^2 Y_1^2(\bar{\mu}_L^-\gamma_\alpha e_L^- + \bar{n}_{32L}^-\gamma_\alpha n_{22L}^-) + \kappa^2 Y_2^2(\bar{n}_{22L}^-\gamma_\alpha n_{12L}^-\\
&+ \bar{\tau}_L^-\gamma_\alpha\mu_L^-) + \kappa^2 Y_1 Y_2(\bar{n}_{22L}^-\gamma_\alpha e_L^-\\
&- \bar{n}_{32L}^-\gamma_\alpha\mu_L^- + \bar{\mu}_L^-\gamma_\alpha n_{12L}^- - \bar{\tau}_L^-\gamma_\alpha n_{22L}^-)\Big]\\
&+ \frac{H_{3L}^{0\alpha}}{\sqrt{2}}\Big[\bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{1L} + \kappa^2 Y_1^2(\bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{eL} - \bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{2L}) + \kappa^2 Y_2^2(\bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{1L}\\
&- \bar{l}_{eL}\sigma_3\cdot\gamma_\alpha l_{\mu L}) - \kappa^2 Y_1 Y_2(\bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{eL} + \bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{\mu L} + \bar{l}_{eL}\sigma_3\cdot\gamma_\alpha l_{2L} + \bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{1L})\Big]\\
&+ H_{4L}^{0\alpha}[\bar{n}_{32L}^+\gamma_\alpha n_{12L}^+ - \kappa^2 Y_1^2(\bar{e}_{32L}^0\gamma_\alpha e_{22L}^0 + \bar{\nu}_{\mu L}\gamma_\alpha\nu_{eL}) - \kappa^2 Y_2^2(\bar{e}_{22L}^0\gamma_\alpha e_{12L}^0\\
&- \bar{\nu}_{\tau L}\gamma_\alpha\nu_{\mu L}) + \kappa^2 Y_2 Y_1(\bar{e}_{22L}^0\gamma_\alpha\nu_{eL} - \bar{e}_{32L}^0\gamma_\alpha\nu_{\mu L} + \bar{\nu}_{\mu L}\gamma_\alpha e_{12L}^0 - \bar{\nu}_{\tau L}\gamma_\alpha e_{22L}^0)\Big]\\
&+ \frac{H_{5L}^{0\alpha}}{\sqrt{2}}\Big[\bar{l}_{3L}^c\cdot\gamma_\alpha l_{2L}^c + \kappa^2 Y_1^2(\bar{l}_{\mu L}\cdot\gamma_\alpha l_{\tau L} - \bar{l}_{3L}\cdot\gamma_\alpha l_{1L}) + \kappa^2 Y_2^2(\bar{l}_{2L}\cdot\gamma_\alpha l_{3L}\\
&- \bar{l}_{\tau L}\cdot\gamma_\alpha l_{eL}) - \kappa^2 Y_1 Y_2(\bar{l}_{3L}\cdot\gamma_\alpha l_{eL} + \bar{l}_{2L}\cdot\gamma_\alpha l_{\tau L} + \bar{l}_{\mu L}\cdot\gamma_\alpha l_{3L} + \bar{l}_{\tau L}\cdot\gamma_\alpha l_{1L})\Big]\\
&+ \frac{H_{6L}^{0\alpha}}{\sqrt{2}}\Big[\bar{l}_{3L}^c\sigma_3\cdot\gamma_\alpha l_{2L}^c + \kappa^2 Y_1^2(\bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{1L} - \bar{l}_{\mu L}\sigma_3\cdot\gamma_\alpha l_{\tau L}) + \kappa^2 Y_2^2(\bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{eL}\\
&- \bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{3L}) + \kappa^2 Y_1 Y_2(\bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{eL} + \bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{\tau L} + \bar{l}_{\mu L}\sigma_3\cdot\gamma_\alpha l_{3L} + \bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{1L})\Big]\\
&+ h.c.\Big\},
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
\mathcal{L}_L^{lA} &= \frac{g}{2\sqrt{2}}\Big\{A_{1HL}^\alpha\Big[\bar{l}_{1L}^c\cdot\gamma_\alpha l_{1L}^c - \bar{l}_{3L}^c\cdot\gamma_\alpha l_{3L}^c + \kappa^2 Y_1^2(\bar{l}_{eL}\cdot\gamma_\alpha l_{eL} - \bar{l}_{3L}\cdot\gamma_\alpha l_{3L})\\
&+ \kappa^2 Y_2^2(\bar{l}_{1L}\cdot\gamma_\alpha l_{1L} - \bar{l}_{\tau L}\cdot\gamma_\alpha l_{\tau L}) + \kappa^2\left(Y_1^2 - Y_2^2\right)(\bar{l}_{2L}\cdot\gamma_\alpha l_{2L} - \bar{l}_{\mu L}\cdot\gamma_\alpha l_{\mu L})
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \kappa^2 Y_1 Y_2 (2\bar{l}_{2L}\cdot\gamma_\alpha l_{\mu L} - \bar{l}_{1L}\cdot\gamma_\alpha l_{eL} - \bar{l}_{3L}\cdot\gamma_\alpha l_{\tau L} + h.c.)\Big] \\
&+ \quad \frac{A^\alpha_{2HL}}{\sqrt{3}}\Big[\ \bar{l}^c_{1L}\cdot\gamma_\alpha l^c_{1L} - 2\bar{l}^c_{2L}\cdot\gamma_\alpha l^c_{2L} + \bar{l}^c_{3L}\cdot\gamma_\alpha l^c_{3L} + \bar{l}_{\mu L}\cdot\gamma_\alpha l_{\mu L} + \bar{l}_{2L}\cdot\gamma_\alpha l_{2L} \\
&+ \quad \kappa^2\left(Y_1^2 - 2Y_2^2\right)(\bar{l}_{eL}\cdot\gamma_\alpha l_{eL} + \bar{l}_{3L}\cdot\gamma_\alpha l_{3L}) - \kappa^2\left(2Y_1^2 - Y_2^2\right)(\bar{l}_{1L}\cdot\gamma_\alpha l_{1L} + \bar{l}_{\tau L}\cdot\gamma_\alpha l_{\tau L}) \\
&+ \quad 3\kappa^2 Y_1 Y_2 (\bar{l}_{3L}\cdot\gamma_\alpha l_{\tau L} - \bar{l}_{1L}\cdot\gamma_\alpha l_{eL} + h.c.)\Big] \\
&+ \quad \frac{A^\alpha_{2AL}}{\sqrt{3}}\Big[2\bar{l}^c_{2L}\sigma_3\cdot\gamma_\alpha l^c_{2L} - \bar{l}^c_{1L}\sigma_3\cdot\gamma_\alpha l^c_{1L} - \bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{3L} + \bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{2L} + \bar{l}_{\mu L}\sigma_3\cdot\gamma_\alpha l_{\mu L} \\
&+ \quad \kappa^2\left(Y_1^2 - 2Y_2^2\right)(\bar{l}_{eL}\sigma_3\cdot\gamma_\alpha l_{eL} + \bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{3L}) \\
&- \quad \kappa^2\left(2Y_1^2 - Y_2^2\right)(\bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{1L} + \bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{\tau L}) \\
&+ \quad 3\kappa^2 Y_1 Y_2 (\bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{\tau L} - \bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{eL} + h.c.)\Big] \\
&+ \quad A^\alpha_{1AL}[\bar{l}^c_{3L}\sigma_3\cdot\gamma_\alpha l^c_{3L} - \bar{l}^c_{1L}\sigma_3\cdot\gamma_\alpha l^c_{1L} + \kappa^2 Y_1^2(\bar{l}_{eL}\sigma_3\cdot\gamma_\alpha l_{eL} - \bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{3L}) \\
&+ \quad \kappa^2 Y_2^2(\bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{1L} - \bar{l}_{\tau L}\sigma_3\cdot\gamma_\alpha l_{\tau L}) + \kappa^2\left(Y_1^2 - Y_2^2\right)(\bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{2L} - \bar{l}_{\mu L}\sigma_3\cdot\gamma_\alpha l_{\mu L}) \\
&+ \quad \kappa^2 Y_1 Y_2 (2\bar{l}_{2L}\sigma_3\cdot\gamma_\alpha l_{\mu L} - \bar{l}_{1L}\sigma_3\cdot\gamma_\alpha l_{eL} - \bar{l}_{3L}\sigma_3\cdot\gamma_\alpha l_{\tau L} + h.c.)\Big]\Big\}, \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{lB}_L \quad = \quad & \frac{g}{\sqrt{2}}\Big\{\frac{B^{-\alpha}_{1L}}{\sqrt{2}}\Big[\bar{e}^{0c}_{12L}\gamma_\alpha n^+_{12L} - \bar{e}^{0c}_{32L}\gamma_\alpha n^+_{32L} + \kappa^2\left(Y_1^2 - Y_2^2\right)(\bar{\mu}^-_L\gamma_\alpha\nu_{\mu L} - \bar{n}^-_{22L}\gamma_\alpha e^0_{22L}) \\
+ \quad & \kappa^2 Y_1^2(\bar{n}^-_{32L}\gamma_\alpha e^0_{32L} - \bar{e}^-_L\gamma_\alpha\nu_{eL}) + \kappa^2 Y_2^2(\bar{\tau}^-_L\gamma_\alpha\nu_{\tau L} - \bar{n}^-_{12L}\gamma_\alpha e^0_{12L}) + \kappa^2 Y_1 Y_2(\bar{e}^-_L\gamma_\alpha e^0_{12L} \\
+ \quad & \bar{n}^-_{12L}\gamma_\alpha\nu_{eL} + \bar{n}^-_{32L}\gamma_\alpha\nu_{\tau L} - 2\bar{\mu}^-_L\gamma_\alpha e^0_{22L} - 2\bar{n}^-_{22L}\gamma_\alpha\nu_{\mu L} + \bar{\tau}^-_L\gamma_\alpha e^0_{32L})\Big] \\
+ \quad & B^{-\alpha}_{2L}\ \Big[\bar{e}^{0c}_{12L}\gamma_\alpha n^+_{22L} + \kappa^2 Y_1^2(\bar{e}^-_L\gamma_\alpha\nu_{\tau L} - \bar{n}^-_{22L}\gamma_\alpha e^0_{12L}) + \kappa^2 Y_2^2(\bar{n}^-_{12L}\gamma_\alpha e^0_{32L} - \bar{\mu}^-_L\gamma_\alpha\nu_{eL}) \\
- \quad & \kappa^2 Y_1 Y_2(\bar{e}^-_L\gamma_\alpha e^0_{32L} + \bar{\mu}^-_L\gamma_\alpha e^0_{12L} + \bar{n}^-_{22L}\gamma_\alpha\nu_{eL} + \bar{n}^-_{12L}\gamma_\alpha\nu_{\tau L})\Big] \\
+ \quad & B^{-\alpha}_{3L}\Big[\bar{e}^{0c}_{12L}\gamma_\alpha n^+_{32L} + \kappa^2 Y_1^2(\bar{e}^-_L\gamma_\alpha\nu_{\mu L} + \bar{n}^-_{22L}\gamma_\alpha e^0_{32L}) + \kappa^2 Y_2^2(\bar{n}^-_{12L}\gamma_\alpha e^0_{22L} + \bar{\mu}^-_L\gamma_\alpha\nu_{\tau L}) \\
- \quad & \kappa^2 Y_1 Y_2(\bar{e}^-_L\gamma_\alpha e^0_{22L} - \bar{\mu}^-_L\gamma_\alpha e^0_{22L} - \bar{n}^-_{22L}\gamma_\alpha\nu_{\tau L} + \bar{n}^-_{12L}\gamma_\alpha\nu_{\mu L})\Big] \\
+ \quad & B^{-\alpha}_{4L}\ \Big[\bar{e}^{0c}_{22L}\gamma_\alpha n^+_{12L} + \kappa^2 Y_1^2(\bar{\tau}^-_L\gamma_\alpha\nu_{eL} - \bar{n}^-_{12L}\gamma_\alpha e^0_{22L}) + \kappa^2 Y_2^2(\bar{n}^-_{32L}\gamma_\alpha e^0_{12L} - \bar{e}^-_L\gamma_\alpha\nu_{\mu L}) \\
- \quad & \kappa^2 Y_1 Y_2(\bar{e}^-_L\gamma_\alpha e^0_{22L} + \bar{\tau}^-_L\gamma_\alpha e^0_{12L} + \bar{n}^-_{32L}\gamma_\alpha\nu_{eL} + \bar{n}^-_{12L}\gamma_\alpha\nu_{\mu L})\Big] \\
+ \quad & B^{-\mu}_{5L}\ \Big[\bar{e}^{0c}_{32L}\gamma_\alpha n^+_{12L} + \kappa^2 Y_1^2(\bar{\mu}^-_L\gamma_\alpha\nu_{eL} + \bar{n}^-_{32L}\gamma_\alpha e^0_{22L}) + \kappa^2 Y_2^2(\bar{n}^-_{22L}\gamma_\alpha e^0_{12L} + \bar{\tau}^-_L\gamma_\alpha\nu_{\mu L})
\end{aligned}
$$

$$
\begin{aligned}
- &\ \kappa^2 Y_1 Y_2(\bar{\mu}_L^- \gamma_\alpha e_{12L}^0 - \bar{\tau}_L^- \gamma_\alpha e_{22L}^0 - \bar{n}_{32L}^- \gamma_\alpha \nu_{\mu L} + \bar{n}_{22L}^- \gamma_\alpha \nu_{eL})\Big] \\
+ &\ \frac{B_{6L}^{-\mu}}{\sqrt{6}}\Big[\bar{e}_{12L}^{0c}\gamma_\alpha n_{12L}^+ - 2\bar{e}_{22L}^{0c}\gamma_\alpha n_{22L}^+ + \bar{e}_{32L}^{0c}\gamma_\alpha n_{32L}^+ - \bar{\mu}_L^- \gamma_\alpha \nu_{\mu L} - \bar{n}_{22L}^- \gamma_\alpha e_{22L}^0 \\
+ &\ \kappa^2\left(2Y_1^2 - Y_2^2\right)(\bar{n}_{12L}^- \gamma_\alpha e_{12L}^0 + \bar{\tau}_L^- \gamma_\alpha \nu_{\tau L}) - \kappa^2\left(Y_1^2 - 2Y_2^2\right)(\bar{n}_{32L}^- \gamma_\alpha e_{32L}^0 + \bar{e}_L^- \gamma_\alpha \nu_{eL}) \\
+ &\ 3\kappa^2 Y_1 Y_2(\bar{e}_L^- \gamma_\alpha e_{12L}^0 + \bar{n}_{12L}^- \gamma_\alpha \nu_{eL} - \bar{n}_{32L}^- \gamma_\alpha \nu_{\tau L} - \bar{\tau}_L^- \gamma_\alpha e_{32L}^0)\Big] \\
+ &\ B_{7L}^{-\mu}\Big[\bar{e}_{22L}^{0c}\gamma_\alpha n_{32L}^+ + \kappa^2 Y_1^2(\bar{n}_{12L}^- \gamma_\alpha e_{32L}^0 - \bar{\tau}_L^- \gamma_\alpha \nu_{\mu L}) + \kappa^2 Y_2^2(\bar{e}_L^- \gamma_\alpha \nu_{\tau L} - \bar{n}_{32L}^- \gamma_\alpha e_{22L}^0) \\
- &\ \kappa^2 Y_1 Y_2(\bar{e}_L^- \gamma_\alpha e_{32L}^0 + \bar{\tau}_L^- \gamma_\alpha e_{22L}^0 + \bar{n}_{32L}^- \gamma_\alpha \nu_{\mu L} + \bar{n}_{12L}^- \gamma_\alpha \nu_{\tau L})\Big] \\
+ &\ B_{8L}^{-\mu}\Big[\bar{e}_{32L}^{0c}\gamma_\alpha n_{22L}^+ + \kappa^2 Y_1^2(\bar{n}_{32L}^- \gamma_\alpha e_{12L}^0 - \bar{\mu}_L^- \gamma_\alpha \nu_{\tau L}) + \kappa^2 Y_2^2(\bar{\tau}_L^- \gamma_\alpha \nu_{eL} - \bar{n}_{22L}^- \gamma_\alpha e_{32L}^0) \\
- &\ \kappa^2 Y_1 Y_2(\bar{\mu}_L^- \gamma_\alpha e_{32L}^0 + \bar{\tau}_L^- \gamma_\alpha e_{12L}^0 + \bar{n}_{32L}^- \gamma_\alpha \nu_{eL} + \bar{n}_{22L}^- \gamma_\alpha \nu_{\tau L})\Big] \\
+ &\ h.c.\Big]. \qquad (32)
\end{aligned}
$$

where we have used the following leptonic doublets: $l_l \equiv (l^-, \nu_l)$ for $l = e, \mu, \tau$ and $l_i \equiv (n_{i2}^-, e_{i2}^0)$ for $i = 1, 2, 3$; and the rotating matrices

$$
\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (33)
$$

6. Stability of the Proton

In the subspace of the fundamental representation of SU$(6)_{CR}\otimes$SU$(6)_{CL}$ the baryon number for G can be associated with the 12×12 diagonal submatrix B=$Diag.[(1/3, 1/3, 1/3, 0, 0, 0) \oplus (1/3, 1/3, 1/3, 0, 0, 0)]$. Since this matrix does not correspond to a lineal combination of generators in G then the baryon number is not gauged in this model (there is not a gauge bosson in G associated with B).

Now due to the stated directions of the VEVs for $\phi_i, i = 1 - 4$ in section 2.3, it is a matter of algebra to show that B$\langle\phi_i\rangle = 0, i = 1 - 4$. Therefore B is not broken spontaneously by the set of Higgs fields used for the breaking of G down to SU$(3)_C\otimes$U$(1)_{EM}$. So, B is perturbatively conserved in the context of the model presented here, and the

proton remains perturbatively stable.

Another way to see this is to use t'Hooft[17] argument and to consider two generators BL and Θ in the subspace of the fundamental representation for SU(6)$_{CR}\otimes$SU(6)$_{CL}$ defined as

$$BL = Diag.[(1,1,1,-1,-1,-1) \oplus (1,1,1,-1,-1,-1)]$$

which is a generator of the G algebra which distinguishes baryon and lepton number, and

$$\Theta = Diag.[(1,1,1,1,1,1) \oplus (1,1,1,1,1,1)]$$

which generates a U(1)$_\Theta$ global symmetry of the model. BL and Θ are spontaneously broken by $\langle\phi_i\rangle, i = 1-4$, but the lineal combination $B = (BL + \Theta)/6$ is not.

7. Concluding remarks

We have studied in detail various aspects of the $[SU(6)]^4 \times Z_4$ grand-unification model, using the fields in the representation $\psi(144) = Z_4\psi(\bar{6},1,1,6)$ as presented in the main text. The most outstanding features of the model are:

- The evolution from low to high energies of the gauge couplings in G, meet together at a single point at the scale $M \sim 10^8$ GeV, in good agreement with precision data tests of the SM. We emphasize that this is the only realistic (small number of low energy Higgs doublets) non supersymmetric model for three families which descends to the SM group in one single step, as a detailed analysis shows [18].

- The low unification scale does not conflict with data on proton stability because baryon number is perturbatively conserved.

- Unlike the model presented in Ref. [12], our $\psi(144)$ does not contain mirror fermions, and it is not vectorlike with respect to G. Therefore the survival hypothesis [2] and

the decoupling theorem [13] can be properly implemented, in such a way that all the exotic fields in $\psi(144)$ get very large masses (of the order of the unification scale).

- At tree level the only ordinary charged fermion field which get mass (of the order M_Z) is the t quark, in consistence with the modified horizontal survival hypothesis [8]. Masses for the other standard charged fermion fields should be generated as radiative corrections.
- The mass terms for the neutral particles of the model show that a generational (three family) see-saw mechanism may easily be implement in order to explain the small neutrino masses [10].

8. Acknowledgments

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APPENDIX A

The terminology used in the main text has been properly translated from classical papers on grand unified theories from fifteen years ago.

Survival Hypothesis[2]. For a symmetry group G with $G' \subset G$, if G is spontaneously broken down to G' at the mass scale M ($G \xrightarrow{M} G'$), then according to the survival hypothesis, any set of fermion fields which are vector representations of G' should get masses of order M. In other words, "at each energy scale the only relevant fermion are those which are chiral with respect to the surviving symmetry".

Extended Survival Hypothesis[15]. It claims that only the scalar fields which acquire VEVs at a particular mass scale, acquire masses at that scale, with the rest of the

scalar fields acquiring masses at the unification scale. In other words, "Higgses acquire the maximum mass compatible with the pattern of symmetry breaking"

Horizontal Survival Hypothesis[4]. It claims that only the particles in the heaviest family of quarks and leptons acquire masses at tree level from dimension four Yukawa couplings, with all the other families getting masses via radiative corrections.

Modified Horizontal Survival Hypothesis[8]. It claims that for a universe with three families, only the top quark and ν_τ acquire tree level masses (the last one lower down with the appropriate see-saw mechanism), with the masses for all the other known fermions generated via radiative corrections.

APPENDIX B

In this appendix we introduce some mathematical definitions used in the main text.

First, the diagonal entries in Eq.(3) are related to the physical fields by

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{pmatrix}_{L(R)} = \begin{pmatrix} 1/\sqrt{6} & 1/2 & 1/\sqrt{12} & 1/2 & 1/\sqrt{12} \\ -1/\sqrt{6} & 1/2 & 1/\sqrt{12} & -1/2 & -1/\sqrt{12} \\ 1/\sqrt{6} & 0 & -2/\sqrt{12} & 0 & -2/\sqrt{12} \\ -1/\sqrt{6} & 0 & -2/\sqrt{12} & 0 & 2/\sqrt{12} \\ 1/\sqrt{6} & -1/2 & 1/\sqrt{12} & -1/2 & 1/\sqrt{12} \\ -1/\sqrt{6} & -1/2 & 1/\sqrt{12} & 1/2 & -1/\sqrt{12} \end{pmatrix} \begin{pmatrix} W^0 \\ A_{1H} \\ A_{2H} \\ A_{1A} \\ A_{2A} \end{pmatrix}_{L(R)} \tag{34}$$

where the gauge fields $W^0_{L(R)}, A_{1HL(R)}, A_{2HL(R)}, A_{1AL(R)}$, and $A_{2AL(R)}$ are related to the following set of diagonal generators of SU(6)$_{L(R)}$:

$Y_{WL(R)} = Diag(1,-1,1,-1,1,-1)/\sqrt{3}; Y_{A_{1HL(R)}} = Diag(1,1,0,0,-1,-1)/\sqrt{2};$

$Y_{A_{2HL(R)}} = Diag(1,1,-2,-2,1,1)/\sqrt{6}; Y_{A_{1AL(R)}} = Diag(1,-1,0,0,-1,1)/\sqrt{2}$, and $Y_{A_{2AL(R)}} = Diag(1,-1,-2,2,1,-1)/\sqrt{6}$, respectively.

The primed fields in Eq.(3) $B'^{\pm}_l, l = 1,6,9$ are related to a set of unprimed ones by the equations

$$\begin{pmatrix} W^{\pm}_{L(R)} \\ B^{\pm}_{1L(R)} \\ B^{\pm}_{6L(R)} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{pmatrix} \begin{pmatrix} B'^{\pm}_1 \\ B'^{\pm}_6 \\ B'^{\pm}_9 \end{pmatrix}_{L(R)} . \tag{35}$$

In order to simplify matters we have defined

$$\begin{pmatrix} H^0_1 \\ H^0_3 \\ H^0_5 \\ H^0_6 \end{pmatrix}_{L(R)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} H'^0_1 \\ H'^0_3 \\ H'^0_5 \\ H'^0_6 \end{pmatrix}_{L(R)} . \tag{36}$$

With the former definitions $W^0_{L(R)}$ and $W^{\pm}_{L(R)}$ are the gauge fields associated with the gauge group SU(2)$_{L(R)}$of the left-right symmetric model.

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